Understanding wave-particle duality through synchronized resonance

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July 23, 2024

Abstract

A photon in superposition can be represented as a Gaussian distribution of frequencies and wavelengths, and therefore, as a population of coupled oscillators. In the classic double-slit experiment, the photon interacts with immobilized atoms in a detector near a slit. This interaction triggers an electron state change in the detector, which emits an electromagnetic wave resonant with the photon's central frequency and wavelength that can reach the photon before it passes. This resonance is hypothesized to pull the photon's states into alignment, synchronizing the population of oscillators and collapsing the superposition into a single frequency and wavelength. Therefore, detector-triggered synchronization of a coupled population of oscillators representing a photon in superposition offers a potential explanation for wavefunction collapse observed in the two-slit experiment.

Part I: Framework

As a generalization of Young's classic double-slit experiment, a photon is found to take multiple paths towards a detection screen representing a state of superposition and resembling a wave when not observed. However, when a detection system is placed near the slits through which the photon may pass, the photon in superposition undergoes wavefunction collapse and behaves as a particle, taking a single path. An example of a common atomic component of detectors near the slits, used to detect the passing photon, is sodium, which can undergo a 3s to 3p electron state transition and releases an omnidirectional electromagnetic wave of approximately 8.21×10^{14} Hz corresponding to a wavelength of ~589 nm[1]. The input photon frequency and wavelength to be measured is selected by its energetic similarity to the energy difference between the atom's electron states and the energy released by the electron state change in the detector, therefore corresponding to a similar frequency and wavelength, $\Delta E = h\nu$. Due to the similarities in their oscillation values, it is possible that the electromagnetic wave released from the electron state change, close to the resonant values of the photon, may drive the synchronization of their oscillations. As the states of the photon are 'coupled' as probabilities, interaction with an electromagnetic wave within the same 'dimension', axis or basis vector as the detector may synchronize photon probabilities into one also within the same 'dimension', axis or basis vector as the detector. If an external electromagnetic wave can interact with a wave in superposition to pull its oscillations into alignment, the probability of detecting the single synchronized wave becomes ~100%, since the superposition of waves of equal oscillation values turns into a single probability, demonstrating particle-like behavior. This can be mathematically modeled by integrating a modified Kuramoto model of oscillator synchronization into an interaction Hamiltonian value that drives distance-dependent oscillator synchronization between frequencies and wavelengths.

Using the relationship between electromagnetic wave frequency ν and wavelength λ, $\nu = \dfrac{c}{\lambda}$ and $\lambda = \dfrac{c}{\nu}$, initial values representing Gaussian distributions of the photon's wavelengths and frequencies as probabilities in superposition can be described as

$$|\psi_A\rangle = \int_{-\infty}^{\infty} \frac{1}{(\pi\sigma^2)^{1/4}} \exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right) |\nu\rangle d\nu$$

in frequency space, and

$$|\psi_A\rangle = \int_{-\infty}^{\infty} \frac{1}{(\pi\sigma^2)^{1/4}} \exp\left(-\frac{(c/\lambda - \nu_0)^2}{2\sigma^2}\right) |\lambda\rangle d\lambda$$

as a wavelength distribution. The photon's central frequency is described by ν_0, while σ is the width of the Gaussian, and the width σ can vary depending on how the photon is generated and released, such as narrowing with the use of a laser (~ 10^7 Hz) or expanding with the use of a lamp (~ 10^{14} Hz).

The full Hamiltonian equation H governing the time evolution of the photon's quantum state through the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

can be written as

$$H = H_A + H_{\text{int}}$$

where $H_A = \hbar\omega_A$ is the Hamiltonian for the photon, with $\omega_A = 2\pi\nu$ and ν is the photon's frequency. This can be related to a time evolution operator $U(t)$, which describes the photon's state over time and can be represented as

$$U(t) = e^{-\frac{i}{\hbar}Ht}$$

The interaction Hamiltonian, H_{int}, describes the influence of electromagnetic field oscillations and synchronization behavior between photon frequency and wavelength probabilities being driven by the external electromagnetic wave emitted from the detector. It borrows from the Kuramoto model,

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$$

where θ_i is the phase of the i-th oscillator, ω_i is the natural frequency of the i-th oscillator, K is the coupling constant and N is the number of oscillators[2]. The interaction Hamilton becomes

$$H_{\text{int}} = gf(d)\sum_i \delta(\nu_i - \nu_0)\cos(\theta_i - \theta_0 - \omega_{\text{ext}}t) + gf(d)\sum_i \delta(\lambda_i - \lambda_0)\cos(\theta_i - \theta_0 - \omega_{\text{ext}}t),$$

where g is the coupling constant, $f(d)$ is the distance-dependent function that can be assumed as 1 for simplicity, $\theta_i - \theta_0$ is the phase difference, $\delta(\nu_i - \nu_0)$ is the Dirac delta function ensuring interaction at specific frequencies, $\delta(\lambda_i - \lambda_0)$ is the Dirac delta function ensuring interaction at specific wavelengths, and ω_{ext} is the external magnetic field oscillation frequency. This frequency, ω_{ext} , representing the electromagnetic oscillation released upon electron state changes in atoms (e.g., ~8.21×10^{14} Hz for 3s to 3p in sodium, and a similar value for the generated photon to be measured via interactions with sodium atoms). The photon's state after a time t due to the interaction Hamiltonian is given by

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi(0)\rangle$$

where $|\psi(0)\rangle$ is the initial state of the photon.

Following Hamiltonian-dependent evolution of the photon state over time, a collapse operator is then created to include the influence of the electromagnetic wave ν_{ext} from the sodium transition, which modifies the evolved state based on the probabilities from the initial Gaussian distribution, as

$$\mathscr{C}(\nu, \nu_0) = \delta(\nu - \nu_0)\exp\left(-\frac{(\nu - \nu_{ext})^2}{2\sigma^2}\right)$$

in frequency space, and

$$\mathscr{C}(\lambda, \lambda_0) = \delta(\lambda - \lambda_0)\exp\left(-\frac{(\lambda - \lambda_{ext})^2}{2\sigma^2}\right)$$

in wavelength space. Application of the collapse operator to the evolved state is represented as

$$|\psi_A\rangle_{\text{final},\nu} = \mathscr{C}(\nu, \nu_0)|\psi(t)\rangle_\nu$$

Upon synchronization and collapse of the traveling photon due to interaction with the electromagnetic wave released from the detector, the photon's final state becomes weighted by the initial Gaussian distribution probabilities with the collapse operator as

$$|\psi_A\rangle_{\text{final},\nu} = \int_{-\infty}^{\infty} \delta(\nu - \nu_0)\exp\left(-\frac{(\nu - \nu_{\text{ext}})^2}{2\sigma^2}\right)\frac{1}{(\pi\sigma^2)^{1/4}}\exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right)|\nu\rangle d\nu$$

in frequency space, and

$$|\psi_A\rangle_{\text{final},\lambda} = \int_{0}^{\infty} \delta(\lambda - \lambda_0)\exp\left(-\frac{(\lambda - \lambda_{\text{ext}})^2}{2\sigma^2}\right)\frac{1}{(\pi\sigma^2)^{1/4}}\exp\left(-\frac{(c/\lambda - \nu_0)^2}{2\sigma^2}\right)|\lambda\rangle d\lambda$$

in wavelength space.

Part II: Change in Probabilities

Using frequency as an example, the probability of the photon being observed at its central frequency is given by

$$P(\nu) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right)$$

using a central frequency of $\nu_0 = 5.09 \times 10^{14}$ Hz and a standard deviation of $\sigma = 10^{13}$ Hz. If calculated within one standard deviation (σ) of the central frequency (ν_0) and given by the integral of the PDF from $\nu_0 - \sigma$ to $\nu_0 + \sigma$, the equation can be represented as

$$P(\nu_0 - \sigma \leq \nu \leq \nu_0 + \sigma) = \int_{\nu_0 - \sigma}^{\nu_0 + \sigma} \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right)d\nu \,.$$

For the standard Gaussian distribution $N(0,1)$, the cumulative distribution function (CDF) evaluated at 1 minus the CDF evaluated at -1 can be represented by the following integral:

$$P(-1 \leq z \leq 1) = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz \,.$$

The variable ν can be converted to the standard normal variable z using the transformation:

$$z = \frac{\nu - \nu_0}{\sigma}$$

therefore

$$P(\nu_0 - \sigma \leq \nu \leq \nu_0 + \sigma) = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz \,.$$

The integral of the standard normal distribution over $[-1,1]$ can be found for the CDF as

$$\Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1$$

where $\Phi(z)$ is the CDF of the standard normal distribution. $\Phi(1)$ can be found to approximately equal 0.8413, so that

$$2\Phi(1) - 1 = 2(0.8413) - 1 = 1.6826 - 1 = 0.6826$$

resulting in a 68.27% probability of the frequency being within one standard deviation of the central frequency. In an example where the detector-emitted electromagnetic wave synchronizes the probability states of the photon into the central frequency, the probability of observing the central frequency becomes 100% and superposition is lost: $P(\nu_0, \text{final}) = 100\,\%$.

Part III: Approximation of the Interaction Distances

To evaluate the possibility of the detector-emitted electromagnetic wave interacting with the passing photon, the speed of light will be approximated as $c \approx 3 \times 10^8$ m/s, and the initial interaction distance of 1 nm will be used as an approximation of the distance for which the traveling photon's electromagnetic field may begin to effectively interact with the electron in the sodium atom. Let us also assume the detector is placed extremely close to the path of the photon but still perpendicular to it. The time required for the photon to cause an electron state change and re-emit an electromagnetic wave can be approximated as the interaction time $t_{\text{int}} \approx 10^{-18}$ s in the magnitude of attoseconds for an approximation of a quantum jump[3]. During this time, the photon would have traveled approximately $(3 \times 10^8 \text{ m/s}) \times (10^{-18} \text{ s})$ $= 3 \times 10^{-10}$ meters $= 0.3$ nanometers. Therefore, after the interaction, the photon is now 0.7 nm (1 nm - 0.3 nm) from the detector when the electromagnetic wave is emitted. Then, let d be the distance traveled by the photon from its current position (0.7 nm from the detector) until it meets the electromagnetic wave. Since both the traveling photon and the emitted wave vibrate omnidirectionally, they will travel towards each other, with the distance traveled by the emitted wave represented as $0.7\,\text{nm} - d$, and since speeds are equal, $2d = 0.7\,\text{nm}$ and $d = 0.35\,\text{nm}$. Therefore, the photon will have only traveled roughly 1/3 of a nm before directly encountering oscillations from the newly emitted electromagnetic wave. Since this is less than the minimum interaction distance, one may assume the waves may meet and interact.

Conclusion

In the classic double-slit experiment, a traveling photon in superposition can interact with atoms in a detector near the slit as it passes by to cause an electron state-change that releases an electromagnetic wave of a similar energy. This emitted wave can in turn reach the traveling photon before it passes the detector. Photon oscillations therefore 'vibrate' the atom into an electron state change which in turn 'vibrate' the photon into a state change. More specifically, the interaction of the external oscillation from the detector with Gaussian oscillation values of the photon is predicted to synchronize their states into one with 100% probability, providing an additional possible explanation for observation-driven wavefunction collapse within the phenomena of wave-particle duality.

References

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